

minimum. The one-particle wave functions (occupied orbitals) from which this Slater determinant is constructed satisfy Hartree-Fock integro-differential equations. The Hartree-Fock equations can be expressed as a one-particle Schrödinger equation for the orbitals with an effective Hamiltonian operator which depends on the set of occupied orbitals but is the same for all of them.

A Slater determinant may be said not to have closed subshells of orbitals when the occupied orbitals, chosen to be basis functions for irreducible representations of some symmetry group, include sets of orbitals which do not completely span irreducible representations. For example, to have closed subshells in $j-j$ coupling, all $2j+1$ different orbitals with different values of the azimuthal quantum number m must be occupied for each given radial wave function.

The theorem proved here states that, for configurations with unfilled shells in the jj -coupling model, the conditions of self-consistency inherent in the definition of the Hartree-Fock one-particle Hamiltonian require that it be of symmetry lower than full spherical symmetry, but are always consistent with spheroidal symmetry. This theorem does not exclude the possibility, in special cases, that the invariance group of the effective Hamiltonian might be some other subgroup of the rotation-inversion group.

Theorem.—If the single Slater determinant of minimum energy in the jj -coupling spherical shell model does not have closed subshells, then there exists a single determinant of lower energy (equal only in case of accidental degeneracy) which has orbitals characterized only by parity and by the azimuthal quantum number. These are eigenfunctions of an axially symmetrical one-particle effective Hamiltonian which is invariant under inversion and includes all exchange effects.

Proof.—By reference 2, the occupied orbitals of a Slater determinant of stationary energy can be expressed as eigenfunctions of a one-particle Hamiltonian which in turn depends on the occupied orbitals through a Hermitian quadratic form. This one-particle Hamiltonian is invariant under all unitary transformations of the occupied orbitals, i.e., under all transformations which carry the corresponding Slater determinant into itself except for a unit phase factor. Hence, since the invariant quadratic combination of orbitals is always unique, the set of occupied orbitals must span a unitary representation of the invariance group of the one-particle Hamiltonian. This is equivalent to the statement that the orbitals fill closed subshells with respect to this group. Since all irreducible representations of the axial group with the inversion operation are one-dimensional, it is always a possible invariance group for the one-particle Hamiltonian corresponding to a single Slater determinant of minimum energy. The full rotation group can satisfy these self-consistent conditions only in the case of closed subshells unless there are accidental degeneracies.

This theorem applies to the Brueckner self-consistent method⁴ as well as to the Hartree-Fock method if the Brueckner method is applied as a Hartree-Fock calculation with a modified two-body interaction. It is assumed that the true many-particle Hamiltonian is spherically symmetrical.

¹ R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. (London) **A70**, 381 (1957); J. Yoccoz, Proc. Phys. Soc. (London) **A70**, 388 (1957); T. H. R. Skyrme, Proc. Roy. Soc. (London) (to be published).

² R. K. Nesbet, Proc. Roy. Soc. (London) **A230**, 312 (1955), Sec. 4; R. K. Nesbet, Phys. Rev. **100**, 228 (1955), Sec. 2.

³ An argument for the spheroidal shell model, based on different assumptions, has been given by J. Rainwater, [Phys. Rev. **79**, 432 (1950)].

⁴ K. A. Brueckner and W. Wada, Phys. Rev. **103**, 1008 (1956); H. A. Bethe, Phys. Rev. **103**, 1353 (1956). Earlier references are given in these two papers.

Interference between Gamow-Teller and Fermi Interaction in Mn^{52}

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THE existence of a strong interference between the Gamow-Teller and the Fermi part of the β interaction has been reported from this laboratory^{1,2} in a study of the circular polarization of the γ rays measured in coincidence with the preceding β particles³ in the case of Sc^{46} . Additional but somewhat weaker evidence has been obtained with similar measurements² on Sc^{44} and V^{48} . Experiments⁴ on Zr^{95} and a reinvestigation⁵ of Sc^{46} have confirmed the existence of such an interference term. On the other hand, the angular distribution of the β particles from polarized neutrons⁶ is not entirely consistent with a large interference term. The existence of the interference has great importance because it rules out combinations of β interactions of the form V , T and S , A and it gives, together with results on electron polarization measurements,⁷ information on the question of time-reversal invariance. The present note describes the result of studies of the interference term in the beta decay of Mn^{52} .

Mn^{52} emits an allowed positron spectrum and three successive γ rays according to the spin pattern 6-6-4-2-0. The β - γ circular-polarization correlation technique as described in reference 2 has been used to find the coefficient A giving the circular polarization of the γ rays. About 0.1 millicurie of carrier-free Mn^{52} was deposited on a 1-mg/cm² mica foil. The difference in coincidence counting-rate for opposite magnetic field directions of the analyzer magnet was found to be $\delta = (-0.30 \pm 0.09\%)$. Corrections have been applied for γ - γ coincidences. The experimental value is based on 18 runs. In each run about 5×10^5 coincidences were collected with alternating magnetic field direction.² The results of the different runs are statistically consistent. The average accepted electron velocity was $0.73v/c$. From the value

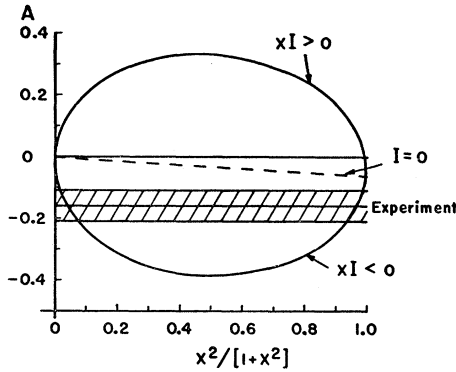


FIG. 1. Anisotropy coefficient A for the circular polarization of γ rays from Mn^{52} . The experimental value of $A = -0.16 \pm 0.05$ is indicated. The theoretical curves correspond to the following different choices of the interference term I and the ratio x between Gamow-Teller and Fermi interaction: $x \cdot I > 0$ and $|I| = \text{maximum}$ (upper branch of the ellipse), $x \cdot I < 0$ and $|I| = \text{maximum}$ (lower branch of the ellipse), and $I = 0$ (dotted straight line). In the case $I = 0$ the maximum theoretical value for the anisotropy is $A = -0.056$. The abscissa represents the percentage of Gamow-Teller contribution. The experimental value indicates the presence of an interference term.

of δ an asymmetry coefficient, $A = -0.16 \pm 0.05$, is derived. A decrease of the circular polarization of the three γ rays due to nuclear recoil effects has not been considered. Our experimental value for $|A|$, therefore, can be considered as a lower limit.

Figure 1 shows the experimental value of A in comparison with theoretical curves derived from the paper by Alder, Stech, and Winther,³ assuming the maximum amount of interference (ellipse) and no interference (dotted straight line). The abscissa represents the percentage of Gamow-Teller contribution. The upper and lower branch of the ellipse hold for positive or negative values, respectively, of the product $x \cdot I$, where x is the ratio of the matrix elements for Gamow-Teller and for Fermi transitions multiplied with the ratio of the respective coupling constants and I is the interference term between both interactions as defined in reference 2. Independent of the unknown ratio of matrix elements, our experimental value indicates the presence of an interference term and shows that $x \cdot I < 0$. The sign of I would follow from this if the sign of the nuclear matrix elements could be predicted.

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² F. Boehm and A. H. Wapstra, Phys. Rev. **109**, 456 (1958).

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⁴ H. Appel and H. Schopper, Z. Physik (to be published).

⁵ R. Steffen (private communication).

⁶ Burgy, Epstein, Krohn, Novey, Raboy, Ringo, and Telegdi, Phys. Rev. **107**, 1731 (1957).

⁷ See, for example, Boehm, Novey, Barnes, and Stech, Phys. Rev. **108**, 1497 (1957); Koller, Schwarzschild, Vise, and Wu, Phys. Rev. **109**, 85 (1958).

Internal Pair Creation in Σ^0 Decay*

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THE existence of a neutral counterpart of the charged Σ hyperons has been established in the past year by several groups.¹ This particle (Σ^0) has been found to decay rapidly (presumably $\tau < 10^{-20}$ sec) into a Λ^0 and photon. It is to be expected that as in the case of other radiative decays, the Σ^0 will exhibit an alternate decay mode in which the photon is internally converted to give an electron pair. One example of such a decay has been found recently by the Columbia bubble chamber group.²

A discussion of the internal pair conversion of high-energy photons has been given by Kroll and Wada,³ who have emphasized that the branching ratio for this process as compared with the radiative decay does not depend strongly on the detailed properties of the system undergoing decay. This conclusion has been found to be essentially valid in the case of Σ^0 decay. However, it may be pointed out that there is some dependence of the result on the relative parity of the Σ^0 and Λ^0 , and that a measurement of the branching ratio

$$\rho = \frac{\Sigma^0 \rightarrow \Lambda^0 + e^+ + e^-}{\Sigma^0 \rightarrow \Lambda^0 + \gamma}$$

with an accuracy of 5% would give some indications of the relative parity of the hyperons.

The effective interaction responsible for the Σ^0 decay is taken to be of the form

$$H_{\text{int}} = g \bar{\psi}_{\Lambda^0} \sigma_{\mu\nu} \psi_{\Sigma^0} F_{\mu\nu} \quad (1)$$

in the case where Λ^0 and Σ^0 have the same parity, and

$$H_{\text{int}} = g' \bar{\psi}_{\Lambda^0} \gamma_5 \sigma_{\mu\nu} \psi_{\Sigma^0} F_{\mu\nu} \quad (2)$$

in the case of opposite parity. It has been assumed that Σ^0 and Λ^0 have spin $\frac{1}{2}$. These are the simplest gauge-invariant interactions which give the decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. It is of course understood that the interactions (1) and (2) may arise through virtual dissociation of the Σ^0 into charged particles, rather than being primary interactions.

With this choice of interactions, the differential transverse and longitudinal conversion factors, R_T and R_L , respectively, which are defined in Eq. (7) of reference 3, are, in the notation of that paper:

$$R_T(x) = 1 - (x^2/E^2), \quad R_L(x) = 0, \quad (3)$$

in the case of the same parity for Σ^0 and Λ^0 , and

$$R_T(x) = 1, \quad R_L(x) = \frac{1}{2},$$

in the case of opposite parity. Here $E = m_{\Sigma} - m_{\Lambda}$, the mass difference of the hyperons and x is an integration